

The electron many-body problem in graphene

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We give a brief summary of the current status of the electron many-body problem in graphene. We claim that graphene has intrinsic dielectric properties which should dress the interactions among the quasiparticles, and may explain why the observation of electron-electron renormalization effects has been so elusive in the recent experiments. We argue that the strength of Coulomb interactions in graphene may be characterized by an effective fine structure constant given by $\alpha^*(\mathbf{k}, \omega) \equiv 2.2/\epsilon(\mathbf{k}, \omega)$, where $\epsilon(\mathbf{k}, \omega)$ is the dynamical dielectric function. At long wavelengths, $\alpha^*(\mathbf{k}, \omega)$ appears to have its smallest value in the static regime, where $\alpha^*(\mathbf{k} \rightarrow 0, 0) \approx 1/7$ according to recent inelastic x-ray measurements, and the largest value in the optical limit, where $\alpha^*(0, \omega) \approx 2.6$. We conclude that the strength of Coulomb interactions in graphene is not universal, but depends highly on the scale of the phenomenon of interest. We propose a prescription in order to reconcile different experiments.

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INTRODUCTION

Graphene is a single atomic layer of graphite, that has been isolated only a few years ago[1], and whose elementary quasiparticle excitations behave as massless Dirac fermions propagating in two spatial dimensions[2]. An important and to the present moment unsolved problem is determining to what extent electron-electron interactions are important in this material. In isolated free standing samples, a trivial estimate for the strength of electron-electron interactions in graphene can be achieved by computing ratio of the Coulomb energy, $E_C = e^2 n^{1/2}$, where e is the electron charge, and n is the electronic density, to the Kinetic energy, $E_K = \hbar v n^{1/2}$, where $\hbar v = 6 \text{ eV\AA}$ is the velocity of the particles. Because the dispersion of the electrons is linear, in two dimensions those two energies scale in the same way with the density, and their ratio is a dimensionless constant known as the fine structure constant[3],

$$\alpha = \frac{e^2}{\hbar v} \approx 2.2, \quad (1)$$

which is 300 times larger than the usual fine structure constant in quantum electrodynamics, $\alpha_{QED} = e^2/(\hbar c) = 1/137$, with c the speed of light.

Although graphene is not as strongly interacting as other materials such as cuprates, where the strength of interactions is measured by the ratio $U/t \sim 10$, where U is the local Coulomb repulsion and t is the hopping energy, by this argument freestanding graphene should not be a weakly interacting system either. The validity of standard perturbation theory requires $\alpha < 1$, which is not applicable in suspended samples. A coupling constant of ≈ 2.2 is large enough to result in a complete breakdown of perturbation theory.

Why graphene fails to exhibit dramatic correlation effects - even in two dimensions - is one of its most challenging puzzles. Exacerbating the problem, a variety of experimental results that are sensitive to interaction effects *appear* to give contradictory results, and further appear to contradict what is expected from theory. Since the Fermi surface of neutral graphene (at half filling) is formed by points at the edges of the Brillouin zone, the Dirac points, metallic screening is not expected to influence its electronic properties due to the vanishing density of states. At the same time, electron-electron interactions are generally expected to give rise to a logarithmic renormalization of the the Fermi velocity at the Dirac points[4],

$$v(q) = v \left[1 + \frac{\alpha}{4} \ln \left(\frac{\Lambda}{q} \right) \right], \quad (2)$$

where Λ is the bandwidth, which plays a role of an ultraviolet cut-off, and q is the momentum measured away from the Dirac point. The structure of perturbation theory in graphene is such that this logarithmic divergence cannot be resummed in higher order expansion in α [4, 5], in contrast to the behavior of Dirac fermions in one dimension, the so called Luttinger liquids[6]. In addition, the electron charge e does not renormalize, so the coupling constant α is expected to be logarithmically renormalized to zero near the Dirac points, i.e. the interaction is marginally irrelevant. Although the damping of the quasiparticles is expected to be $\tau(\omega_0) \propto \alpha^2 |\omega_0|$ to leading order in α , where $|\omega_0| = \hbar v q$ is the energy of the quasiparticles, because of the logarithmic renormalization of v and α the ratio $\tau(\omega_0)/|\omega_0| \ll 1$ is small in the limit $|\omega_0| \rightarrow 0$, so the quasiparticles are still well defined (with logarithmic accuracy) near the Dirac points. In general, the renormalization of all physical quantities,

such as the compressibility, susceptibilities, etc. can be derived directly from their scaling dependence with the renormalized velocity[7–9].

To be more concrete, one should expect for instance the single-particle spectrum of graphene, to be logarithmically renormalized near the Dirac points. In supported samples, the fine structure constant is dressed by dielectric screening effects from the substrate, $\alpha = e^2/(\hbar v \epsilon_0)$, with ϵ_0 the dielectric constant. Angle resolved photoemission spectroscopy (ARPES) experiments do not show any evidence of a logarithmic renormalization in the spectrum[10], even though in graphene on a moderate dielectric substrate such as SiO_2 , where $\alpha \sim 1$, a visible renormalization should be expected in the energy range of eV away from the Dirac point.

Another interesting observable that can reveal renormalization effects in graphene is the electronic compressibility, $\kappa = (\partial^2 E / \partial n^2)^{-1}$, with E the total energy, which measures the strength of interactions in an electron gas. This measurement has been recently carried out by a single electron transistor (SET) experiment[11], which revealed large puddles of change in graphene at densities as low as 10^{11}cm^{-2} , which corresponds to a Fermi level ~ 50 meV away from the Dirac point. For non-interacting Dirac fermions, $\kappa^{-1} = v\sqrt{\pi}/4n$. Many body effects were expected to give an additional logarithmic correction with the density due to the velocity renormalization[8]. The deviation from the non-interacting result, nevertheless, was not observed. Although those experiments were not carried out on suspended samples, the modest dielectric screening from the SiO_2 substrate is not sufficient to explain the absence of verifiable renormalization effects. To make things more intriguing, recent ARPES measurements on doped graphene samples observed a non-trivial splitting of the Dirac cones[12], which were attributed to the formation of plasmarons, a composite particle formed by an electron and a plasmon, the collective charge excitation of the Fermi sea.

Many-body effects have also been observed in the presence of strong magnetic fields[13], which quench the kinetic energy into discrete Landau levels. The most convincing evidence of interactions is based in the observation of $\nu = 1/3$ plateau in the fractional quantum hall effect in suspended samples[14–16] and in samples supported on boron nitride[17]. At zero magnetic fields, on the other hand, there is no experimental evidence so far of the excitonic gap[18], which has been predicted to open up in graphene when $\alpha > 1.1$ [19]. In all current transport and spectroscopy experiments, graphene seems to behave as a semi-metal[20, 21].

From the point of view of Coulomb scatterers, experiments involving the adsorption of K adatoms in graphene have reported a significant change in the electronic mobility of the samples[23], what was interpreted as an indication that the transport in suspended samples was severely influenced by Coulomb scattering from charge

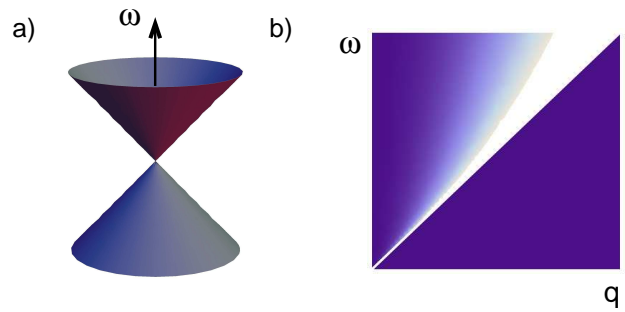


Figure 1: a) Energy spectrum of the electrons around the Dirac point, $\omega = \hbar v q$. b) Density map of the imaginary part of the polarization for non-interacting Dirac fermions, $\text{Im}\Pi^{(1)}(q, \omega)$. Dark blue color: $\text{Im}\Pi^{(1)}(q, \omega) = 0$; white region: optical absorption edge at the border of the particle-hole continuum ($\omega > \hbar v q$).

impurities. This interpretation is nevertheless at odds with another experiment[22] in which the dielectric constant of the SiO_2 substrate was enhanced up to two orders of magnitude by embedding the substrate in highly dielectric fluids. The variation of the mobility in this experiment was no more than 30%, indicating that Coulomb impurities do not influence the electronic properties of graphene.

As we argue below, these apparent inconsistencies among different experiments and the difficulty of observing electronic many body effects in the various physical observables may be due to the intrinsic dielectric screening properties of graphene itself. In suspended samples, where $\alpha = 2.2$, interactions among the electrons can be quite strong and lead to dynamical screening of the quasiparticles, which can be much more weakly interacting than previously believed.

THE POLARIZATION OF THE VACUUM

As the electrons interact, virtual processes that excite electrons from the filled valence band up to the empty conduction band spontaneously create particle-hole pairs which eventually recombine and decay back into the ground state, ie. the vacuum. The process of spontaneous creation and annihilation of particles and holes polarizes the charge of the system. This polarization is expected to dress the quasiparticles and give rise to screening. Due to the absence of a Fermi surface (say at half-filling), the screening is dielectric in nature, in the sense that the Coulomb interaction remains long ranged but parametrically weaker. For a review about electron-electron interactions and the charge polarization of graphene, see Ref.[3].

Although graphene is two dimensional, the fact that it is a semi-metal rather than gapped allows dielectric screening to emerge at long wavelengths. This is eas-

ily understood if one computes the charge polarization function for non-interacting Dirac fermions in 2 + 1 dimensions,

$$\Pi^{(1)}(q, \omega) = -\frac{1}{4} \frac{q^2}{\sqrt{(\hbar v q)^2 - \omega^2}}. \quad (3)$$

This polarization function is strictly real for $\omega < \hbar v q$ and becomes purely imaginary for $\omega > \hbar v q$, in the region of the particle-hole continuum where virtual excitation processes from the lower to the upper band are allowed (see Fig. 1). In the static regime ($\omega = 0$), the dielectric function of graphene in random phase approximation (RPA) is

$$\epsilon(q, 0) = 1 - V(q)\Pi^{(1)}(q, 0) = 1 + \pi\alpha/2 \approx 4.45 \quad (4)$$

for $\alpha = 2.2$ [24], where

$$V(q) = \frac{2\pi e^2}{q} \quad (5)$$

is the Fourier transform of the Coulomb interaction $V(r) = e^2/r$ in 2D. For gapped graphene, as in any semiconductor, there is a crossover in the behavior of the static polarization function at long wavelengths, $\hbar v q \ll \Delta$, $\Pi^{(1)}(q, 0) = -q^2/\Delta$, where Δ is the energy gap[25]. In two spatial dimensions, where the Coulomb interaction $V(q) \propto 1/q$, one recovers the dielectric constant of the vacuum $\epsilon(q \ll \Delta/\hbar v, 0) = 1$, in contrast with 3D insulators ($V(q) \propto 1/q^2$), where the dielectric function saturates to a constant for $q \ll \Delta/\hbar v$.

In the gapless case, if $\alpha = 2.2$, there is no reason, a priori, to trust in the leading correction of perturbation theory in the physical observables, and one should seek to include diagrams of all orders in α . If one goes on to include the next correction to the dielectric constant, which is enclosed in the vertex correction of the bubble, $V(q)\Pi_{\text{vertex}}^{(2)}(q, 0) \approx -0.53\alpha^2$. For $\alpha = 2.2$, this term is almost of the same order of the RPA correction, in which case[26]

$$\epsilon(q, 0) = 1 + \pi\alpha/2 + 0.53\alpha^2 \approx 7 + O(\alpha^3). \quad (6)$$

In the opposite regime, where $\omega \gg \hbar v q$, one recovers the gapped situation, since the polarization bubble in leading order is

$$\Pi^{(1)}(q \rightarrow 0, \omega) = -\frac{1}{4} \frac{q^2}{i\omega}, \quad (7)$$

and hence $\epsilon(q \rightarrow 0, \omega) \rightarrow 1$. Therefore, the screened Coulomb interaction, and as a consequence the dressed fine structure constant of graphene,

$$\alpha^*(q, \omega) \equiv \frac{e^2}{\hbar v \epsilon(q, \omega)}, \quad (8)$$

has two distinct limits: the static regime, where $\alpha^*(q, 0) = \alpha_G < 2.2$, and the dynamic limit, where

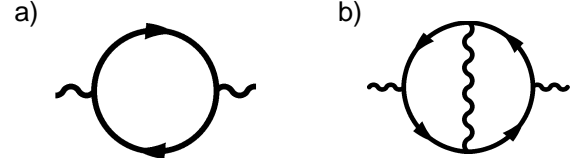


Figure 2: Diagrammatic representation of a) the leading term in the polarization function, $\Pi^{(1)}$ and b) the lowest order vertex correction.

screening is ineffective and $\alpha^*(0, \omega) = 2.2$ recovers the bare coupling constant of freestanding samples. In the on-shell intermediate case, where $\omega \approx \hbar v q$, the leading vertex correction to the polarization bubble has a logarithmic divergence near the on-shell region[27],

$$\Pi_{\text{vertex}}^{(2)}(q, \omega) \propto \frac{q^2}{\omega - \hbar v q} \ln\left(\frac{\hbar v q}{|\omega - \hbar v q|}\right). \quad (9)$$

Resummation of this divergence in the ladder channels to all orders in α has been proposed to give rise to a zero of the dielectric function corresponding to an excitonic bound state that lives in the optical gap of the particle-hole continuum[27]. In any case, the RPA approximation seems to underestimate screening in the low energy sector $\omega \ll \hbar v q$ and does not account for excitonic effects at the edge of the particle-hole continuum.

From the point of view of Coulomb impurities, the amount of charge induced by the polarization of the vacuum around a test charge Q can be computed directly from the dielectric function assuming linear response,

$$Q_{\text{induced}}(q) = -Q \left(1 - \frac{1}{\epsilon(q, 0)}\right), \quad (10)$$

where $\epsilon(q, \omega) = 1 - V(q)\Pi(q, \omega)$ is the dielectric function. Since for Dirac fermions $\epsilon(q, 0)$ is a constant, the real space distribution of the induced charge is a delta function centered at the impurity. For $\alpha = 2.2$, the overall induced charge is $Q_{\text{induced}}^{\text{RPA}} \approx -0.77Q$ in RPA, and $Q_{\text{induced}} \approx -0.86Q + O(\alpha^3)$ when the first vertex correction in the polarization is included.

EXPERIMENTAL MEASUREMENT OF THE POLARIZATION

One experiment that can provide direct information about the charge polarization properties of a system is the measurement of the response function with inelastic x-ray scattering. The response function is the imaginary part of the charge susceptibility, $\chi(q, \omega)$, which is defined in terms of the full polarization function, $\Pi(q, \omega)$,

$$\chi(q, \omega) = \frac{\Pi(q, \omega)}{\epsilon(q, \omega)}. \quad (11)$$

One major difficulty with this experiment for free standing graphene samples is that x-ray experiments cannot readily be performed on a one atom thick material. This difficulty can be overcome by realizing that although the susceptibility of the multilayer system can be quite different from the single layer case, they have very similar polarization functions at momentum and energy scales that are much larger than the electronic hopping energy between the different layers. The argument is the following: let us suppose for the moment that the interlayer hopping energy, t_{\perp} , is zero. The Coulomb interaction is long ranged and couples all the layers,

$$V_{3D}(k) = \frac{4\pi e^2}{|\mathbf{k}|^2 + k_z^2}, \quad (12)$$

with $\mathbf{k} = (k_x, k_y)$ the in-plane momentum. In the diagram in Fig. 2b for the vertex, it is clear that since the fermionic lines have no k_z dependence (since the fermions do not disperse along that direction when $t_{\perp} = 0$), the integration over k_z in the internal Coulomb line gives $\int_{-\infty}^{\infty} \frac{dk_z}{2\pi} V_{3D}(k) = 2\pi e^2/|\mathbf{k}|$, which is the Coulomb interaction of electrons in the single layer. This argument can be extended to all orders in perturbation theory in α . Hence, the polarization functions of the freestanding single layer and of the multilayer systems should be identical provided they share the same single particle energy spectrum[30].

Since the external momentum and frequency enter as an infrared cut-off of the momentum integrals in the polarization, provided that t_{\perp} is much smaller than this energy scale, say $t_{\perp} \ll \max(\hbar v q, \omega)$, t_{\perp} can only give subleading corrections to the polarization but cannot affect the leading term. The similarity between the polarization function of graphene and the multi-layer case breaks down if $t_{\perp} \sim \max(\hbar v q, \omega)$, in which case t_{\perp} becomes the infrared cut-off itself, giving rise to a crossover. The same rationale applies to other possible extrinsic infrared energy scales, such as a Fermi pocket, E_F , which shifts the chemical potential away from the neutrality point, or an external magnetic field, B . Both are expected to give rise to a crossover in the behavior of the dielectric function at sufficiently low energy scales, but do not affect the leading term of the dielectric function provided $E_F, B \ll \max(\hbar v q, \omega)$. Hence, in spite of the fact that the dimensionality of the Coulomb interaction is different in the freestanding single layer and in the multilayer case, the polarization of the vacuum is quite similar in those two systems at energy scales where the single particle spectrum of the two is essentially the same.

Examples of multilayer graphene systems include graphite in the Bernal AB stacking and other variations including turbostratic graphite, where the layers are randomly rotated. Information about the vacuum polarization of the single freestanding layer can be also obtained experimentally from samples with a finite number of layers. In single crystals of graphite, $t_{\perp} \approx 0.39\text{eV}$ is the

interlayer hopping energy, below which the bands have hyperbolic dispersion near the K points of the Brillouin zone. In the turbostratic case, t_{\perp} can be much smaller since the coherence of the hopping between different layers is suppressed. Multilayer epitaxial grown graphene samples could also be suitable for x-ray experiments, and reveal detailed information about the vacuum polarization properties of isolated graphene in the infrared.

Recent inelastic x-ray experiments in single crystals of graphite conducted by the authors[28] have revealed that freestanding graphene is a highly polarizable system. Fig. 3 summarizes the result of those measurements. Fig. 3a shows the absolute value of the screened fine structure constant for a freestanding layer, which ranges from $\alpha^*(q, 0) = \alpha_G \approx 0.142 \pm 0.092 \approx 1/7$ in the static regime ($\omega \ll \hbar v q$) and long wavelengths, where screening is more effective, up to $\alpha^*(q, \omega) \approx 2.6$ in the opposite limit, $\omega \gg \hbar v q$, where screening is weak and collective modes such as plasmons emerge due to the diverging density of states near the Brillouin zone boundary. The solid line $\omega = \hbar v q$ is a guide to the eye, and indicates the edge of the particle hole continuum in the region of the Dirac cone ($q < 0.5 \text{\AA}^{-1}$). In panel 3c the static charge susceptibility it is shown as a function of the momentum. This figure indicates that the leading behavior of $\chi(q, 0)$ is linear in q , which is expected for a system with Dirac fermions and results in a finite screening strength at long distances.

The imaginary part of the polarization function for the two lowest momentum data points, $\text{Im}\Pi(q, \omega)$, is shown in Fig. 3b as a function of frequency and is compared with the imaginary part of the polarization for non-interacting Dirac fermions, $\text{Im}\Pi^{(1)}(q, \omega)$ (solid lines), defined by Eq. (3). The peak in the solid curves indicates the optical adsorption edge of the particle-hole continuum for a fixed momentum, depicted in Fig. 1b. The data points show a redshift of the optical adsorption edge in $\sim 0.6\text{eV}$, which is similar in magnitude with the result of a recent *ab initio* calculation that explicitly accounted for interactions in the particle-hole channel[29]. This redshift is interpreted as an excitonic shift in spectral weight due to Coulomb interaction between electrons and holes in the conduction and valence bands respectively. Hence, in spite of being a 2D semi-metal, graphene is polarizable to a degree similar to a conventional, 3D semiconductor such as Si or GaAs.

Another way to view the significant, static polarizability of graphene is in the screening of a test charge. The reconstruction of the induced charge density in linear response theory gives $Q_{\text{induced}} = -(0.924 \pm 0.046)Q$ and the net impurity charge is only a small fraction of the charge of the original perturbation. In real space, the spatial distribution of the density is a local cloud of charge with a core of approximately $R_0 \sim 10\text{\AA}$ of size, which quickly disappears beyond 15\AA away from the impurity [28]. Since the Dirac Hamiltonian does not

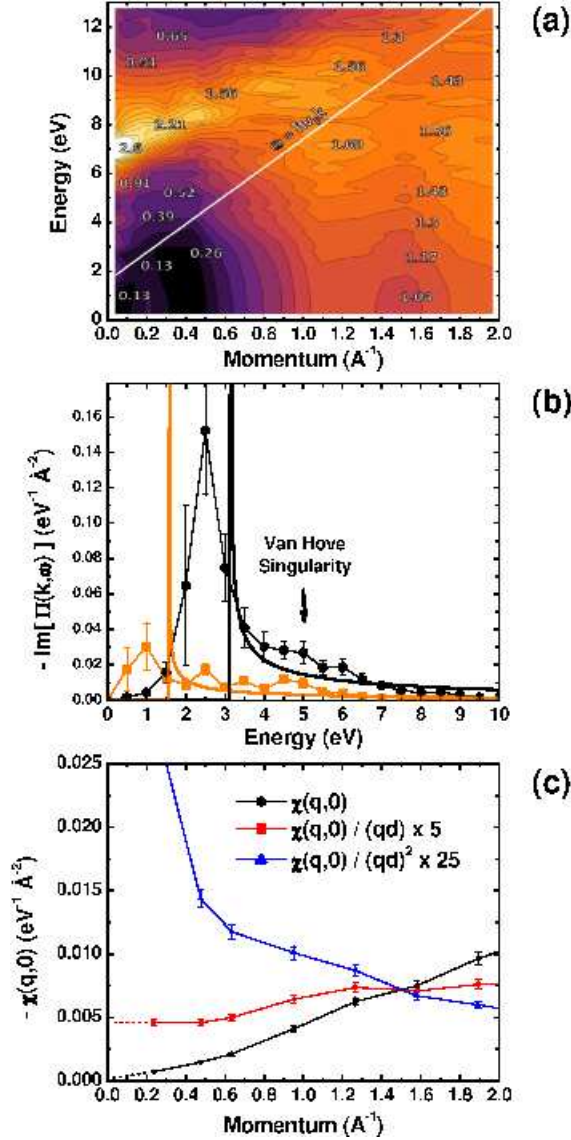


Figure 3: (a) The magnitude of the effective, screened fine structure constant, $\alpha_g^*(k, \omega)$, reproduced from ref. [28]. (b) Imaginary part of the polarization function, $\Pi(q, \omega)$, measured with inelastic x-ray scattering, compared with what is expected for ideal, Dirac Fermions (eq. 3). (c) Asymptotic properties of the static, charge response function, $\chi(q, 0)$, showing linear, leading behavior at small q .

have a length scale, the corresponding charge response is purely local, as discussed below Eq. (10). The high energy states by their turn affect the physics locally at length scales of the order the lattice spacing and cannot not directly influence the screening properties of the system at long distances. Therefore, in half-filled graphene the induced charge should be completely confined in a finite-sized region around the impurity. The implication is that the problem of screening a Coulomb impurity in graphene should manifest a complete separation of length scales. Any crossover induced by some infrared energy

scale E_0 , such as a small Fermi surface pocket, which restores metallic screening, or a small energy gap which leads to insulating behavior, will be manifested only beyond length scales of the order of $\lambda \sim \pi \hbar v / E_0$. For a small energy scale of $E_0 = 0.1 \text{ eV}$ measured away from the Dirac point, the corresponding length scale is of the order of $\lambda \sim 200 \text{\AA}$. Hence, in between these two different length scales, namely R_0 and $\lambda \gg R_0$, the behavior of graphene is quite universal and screens the test charge nearly completely.

The value of the net charge found in the inelastic x-ray experiment suggests that graphene has an intrinsic static dielectric constant of the order of 10 [$\epsilon(k, 0) \approx 15$], which of course is just another way of visualizing the value given in the $\omega = 0, q \rightarrow 0$ region of Fig. 3a. This result implies that the transport is not significantly affected by Coulomb scatterers[22]. Although potassium atoms change substantially the electronic mobility in graphene[23], the scattering does not seem to be driven by the long range part of the Coulomb interaction, but rather by the short range part of it.

PREScription FOR PHYSICAL OBSERVABLES

In suspended samples, the expansion in the unscreened $\alpha = 2.2$ is poorly controlled,. The calculation of various physical observables such as the self-energy of the electrons and the total energy, from which one can extract the compressibility, can be instead organized in powers of the dressed Coulomb interaction, in the hope that the screening of the interactions among the quasiparticles will result in a convergent expansion.

The use of the dynamically screened Coulomb interaction in the calculation of all physical observables may for instance reconcile apparent contradictions between different kinds of experiments. For instance, the existence of plasmarons in graphene, which were proposed to exist in a region of the phase space where screening is typically weak ($\omega \gg \hbar v q$), is not inconsistent with the observation that Coulomb impurities can be strongly screened out by the polarization of the vacuum, which is related to the static response of the system ($\omega \ll \hbar v q$), where screening is strong. The observation of the fractional quantum Hall effect is not inconsistent either with the idea that graphene is a strongly polarizable medium: the magnetic field in this case plays the role of an infrared cut-off that sets an energy scale below which the dielectric function undergoes a crossover in direction to a “gapped state”. At energies comparable or smaller than this energy scale, the vacuum is strongly reconstructed by the Landau levels, which quench the kinetic energy and make interactions stronger.

The lack of observable logarithmic renormalization effects in the SET experiments on the compressibility in

the single layer supported on a SiO₂ substrate and also in ARPES experiments, can be attributed to dynamical screening effects, which inhibit the velocity renormalization at the energy scales currently reached by most experiments. Very recently, the measurement of Shubnikov-deHaas oscillations at very low magnetic fields ($B \sim 0.01\text{T}$) and densities as low as 10^9cm^{-2} have revealed the first indirect observation of the logarithmic renormalization of the electronic spectrum in graphene[31]. An RG calculation incorporating dynamically generated screening within the RPA approximation yielded qualitative agreement with experiment. How to resolve this observation with x-ray experiments, in which the breakdown of RPA is very clear (Fig. 3b), is not immediately obvious. The assignment of an effective value for α in the Shubnikov-deHaas experiment ($\alpha_G \approx 0.6$ [29]) reflects an average over the strength of the Coulomb interactions in the entire dynamical range, and is not in principle inconsistent with the boundary values of α ($1/7$ in the static case and 2.6 in the optical regime) found in the x-ray experiment. It is perhaps suggestive that this effective value of $\alpha_G \approx 0.6$ approximately coincides with the value of the dressed fine structure constant at the edge of the particle-hole continuum [see Fig. 3 (a)], where on shell processes should dominate the renormalization of the spectrum. Further infrared experiments are needed to verify the actual level of agreement between those two experiments.

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situation is given by $V_{3D}(k) = 2\pi e^2 d S(k)/|\mathbf{k}|$, which is the Fourier transform of the e^2/r interaction for an infinite stack of layers separated by a distance d , and $S(k) = \sinh(|\mathbf{k}|d)/[\cosh(|\mathbf{k}|d) - \cos(k_z d)]$ is the structure factor. Integration over k_z in the Brillouin zone of size

$2\pi/d$, namely $\int_{-\pi/d}^{\pi/d} \frac{dk_z}{2\pi} V_{3D}(k) = 2\pi e^2/|\mathbf{k}|$, which again is the Coulomb interaction in 2D.
 [31] D. C. Elias *et al.*, arXiv:1104.1396(2011).